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Game Technology
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Solution 9

For bonus points upload your solutions until **Tuesday, January 9th, 2018, 13:29**

General Information

- The exercises may be solved by teams of up to three people.
- The solutions have to be uploaded to the Git repositories assigned to the individual teams.
- **The submission date (for practical and theoretical tasks) is noted on top of each exercise sheet.**
- If you have questions about the exercises write a mail to game-technology@kom.tu-darmstadt.de or use the forum at https://www.fachschaft.informatik.tu-darmstadt.de/forum/viewforum.php?f=557

P9 Practical Tasks: Physics (5 points)

In this exercise, the overall task is to build a simple version of "Marbellous". The extended physics code which handles collisions between the ball and the triangle mesh are provided for the most parts.

The code is provided for you, your task is to fill out the respective functions. The code can be found at https://github.com/TUDGameTechnology/Exercise9.git

Please remember to push into a branch called "exercise9".

To get the solution source code, please clone https://github.com/TUDGameTechnology/Solution9.git.

P9.1 Triangle-Sphere-Intersection (2 points)

In `Collision.h`, you can find the source code for the SAT intersection test for triangles and spheres. (Note that the remaining code is an optimized version of the test). Provide the code for the functions `IsSeparatedByA`, `IsSeparatedByB` and is `IsSeparatedByC` which should be true iff the axis from the vertex a, b or c to the sphere is a separating axis.

P9.2 Sphere-Box-Intersection (3 points)

(See also the theoretical task.) Implement your box-sphere-intersection algorithm. Use it to detect when the ball has reached the goal area. Play the provided sound when the goal area is reached.

The goal is a box (rectangular cuboid) that is centered at the point (x, y, z) = (-46, -4, 44). The full extents of the sides of the box are approximately (10.6, 4.4, 4.0). The box is not rotated and therefore aligned to the global coordinate system.

T9 Theoretical Task: Physics (5 points)

T9.1 Sphere-Plane-Collision (2 points)

Consider the situation below, which gives you information about the current state of a sphere and an immovable plane. You can disregard gravity for this exercise.

Plane: $\boldsymbol{d} = \mathbf{1}, \boldsymbol{n} = \begin{pmatrix} \mathbf{0} \\ \mathbf{1} \\ \mathbf{0} \end{pmatrix}$

Circle: $\text{radius} = \mathbf{3}, \text{position} = \begin{pmatrix} \mathbf{5} \\ \mathbf{2} \\ \mathbf{1} \end{pmatrix}, \text{velocity} = \begin{pmatrix} \mathbf{-0.5} \\ \mathbf{1} \\ \mathbf{1} \end{pmatrix}$

a) Calculate the following values. Please include your calculations.

Distance (sphere center to plane):

Collision normal:

Penetration depth:

Separating Velocity:

Distance: *We know that we can get the distance of a point to a plane by entering the point into the equation of the plane:*

$$\begin{aligned} \boldsymbol{D} &= \boldsymbol{xn} - \boldsymbol{d} \\ &= \begin{pmatrix} \mathbf{5} \\ \mathbf{2} \\ \mathbf{1} \end{pmatrix} \begin{pmatrix} \mathbf{0} \\ \mathbf{1} \\ \mathbf{0} \end{pmatrix} - \mathbf{1} \\ &= \mathbf{2} - \mathbf{1} \\ &= \mathbf{1} \end{aligned}$$

Collision normal: *The collision normal is the same as the normal of the plane:*

$$\begin{pmatrix} \mathbf{0} \\ \mathbf{1} \\ \mathbf{0} \end{pmatrix}$$

Penetration depth: *The penetration depth is the part of the sphere that is inside the plane. We get this by comparing the distance of the sphere center to the plane and the sphere radius:*

$$\boldsymbol{r - d = 3 - 1 = 2}$$

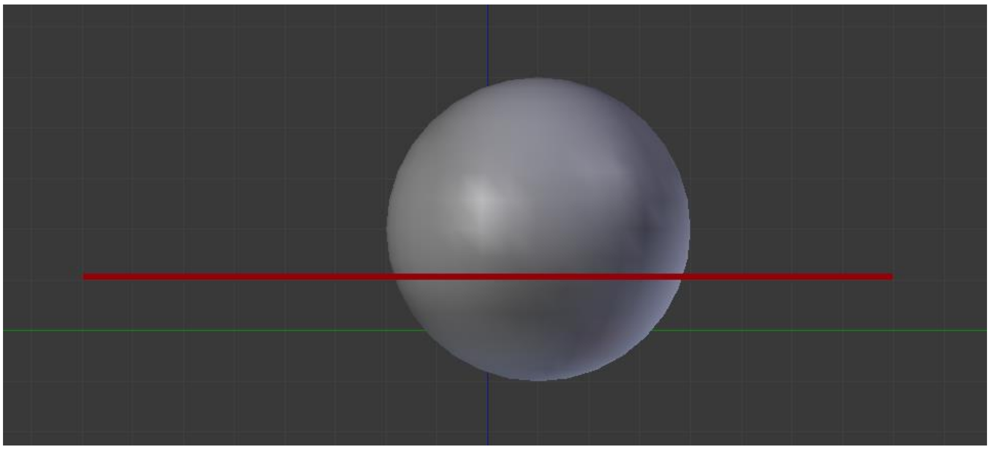
Separating velocity: *To get the separating velocity, we need to consider the projection of the sphere's velocity onto the collision normal. We do not need to use two velocities since the plane is not moving.*

$$\begin{pmatrix} \mathbf{-0.5} \\ \mathbf{1} \\ \mathbf{1} \end{pmatrix} \begin{pmatrix} \mathbf{0} \\ \mathbf{1} \\ \mathbf{0} \end{pmatrix} = \mathbf{1}$$

b) Are the sphere and the plane colliding? Explain your answer.

Yes, they are. We can read this from the penetration depth: It is positive, therefore, the sphere and the plane are intersecting.

You can also see this in the 3D rendering of the scene:



c) Should we apply some collision response? Explain your answer. (You don't need to specify which collision response, if any is required).

Since the separating velocity is > 0, the objects are already separating, so we don't need to separate them.

We might reset the sphere to be outside of the plane immediately, as described in the lecture slides on slides 45 – 46.

T9.2 Separating Axis (3 Points)

Consider the following situation with a rectangle and a triangle.

Provide an axis that is a separating axis for these two objects and explain why your axis is separating the objects, using the definition from the lecture. The rectangle is not rotated.

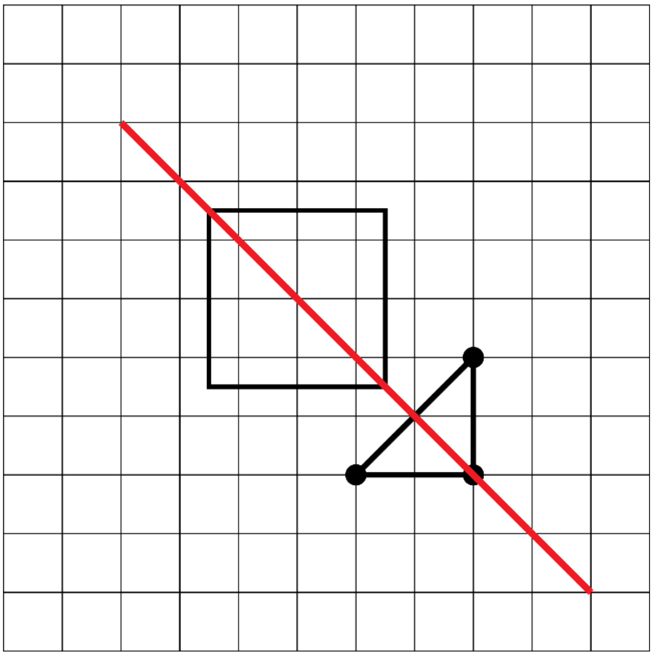
Specify the separating axis by providing a point and a normal direction.

Note: You may of course make a diagram to help you visualize the exercise, but answers using only diagrams are not counted.

Rectangle (not rotated)	
Center	$\begin{pmatrix} \mathbf{5} \\ \mathbf{6} \end{pmatrix}$
Edge Lengths	$\begin{pmatrix} \mathbf{3} \\ \mathbf{3} \end{pmatrix}$
(Vertices)	$\{\begin{pmatrix} \mathbf{3.5} \\ \mathbf{4.5} \end{pmatrix}, \begin{pmatrix} \mathbf{6.5} \\ \mathbf{4.5} \end{pmatrix}, \begin{pmatrix} \mathbf{3.5} \\ \mathbf{7.5} \end{pmatrix}, \begin{pmatrix} \mathbf{6.5} \\ \mathbf{7.5} \end{pmatrix}\}$
Triangle	
Vertices	$\{\begin{pmatrix} \mathbf{6} \\ \mathbf{3} \end{pmatrix}, \begin{pmatrix} \mathbf{8} \\ \mathbf{5} \end{pmatrix}, \begin{pmatrix} \mathbf{8} \\ \mathbf{3} \end{pmatrix}\}$

Axis	
Point	$\begin{pmatrix} \mathbf{5} \\ \mathbf{6} \end{pmatrix}$
Normal	$\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\begin{pmatrix} \mathbf{1} \\ \mathbf{-1} \end{pmatrix}$

If we look at the diagram of this configuration, we see that the main axes x and y are no candidates for a separating axis because their projections onto the axes overlap.



We find one axis that is simple to use: The axis that is orthogonal to the edge of the triangle that points towards the rectangle.

We see two points of the rectangle and one point of the triangle are already on the axis. We define the axis to start at the top-left point of the rectangle and point towards $\begin{pmatrix} \mathbf{1} \\ \mathbf{-1} \end{pmatrix}$.

The distances of the points of the rectangle are then defined in terms of the diagonal $\boldsymbol{d}$ *of the square.*

$$\boldsymbol{d} = \sqrt{\mathbf{3^2 + 3^2}} = \sqrt{\mathbf{18}} \approx \mathbf{4.24}$$

$$\left\{\mathbf{0}, \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{d}, \boldsymbol{d}\right\}$$

Therefore, $\boldsymbol{min_{square}} = \mathbf{0}, \boldsymbol{max_{square}} = \mathbf{4.24}$.

To find the projection of the two vertices of the triangle that do not lie on the axis already, we need to find their median point, since the triangle is symmetric around the axis:

$$\boldsymbol{p} = \frac{\mathbf{1}}{\mathbf{2}}\left(\begin{pmatrix} \mathbf{6} \\ \mathbf{3} \end{pmatrix} + \begin{pmatrix} \mathbf{8} \\ \mathbf{5} \end{pmatrix}\right) = \frac{\mathbf{1}}{\mathbf{2}}\begin{pmatrix} \mathbf{14} \\ \mathbf{8} \end{pmatrix} = \begin{pmatrix} \mathbf{7} \\ \mathbf{4} \end{pmatrix}$$

The distance of this point from the start point of our axis is:

$$\left|\begin{pmatrix} \mathbf{3.5} \\ \mathbf{7.5} \end{pmatrix} - \begin{pmatrix} \mathbf{7} \\ \mathbf{4} \end{pmatrix}\right| = \left|\begin{pmatrix} \mathbf{-3.5} \\ \mathbf{3.5} \end{pmatrix}\right| \approx \mathbf{4.95}$$

We know that the other vertex of the triangle is further away than this point. Therefore we, can already determine that this is a separating axis:

$$\boldsymbol{max_{square}} < \boldsymbol{min_{triangle}}$$
$$\mathbf{4.24} < \mathbf{4.95}$$

We see that the condition on slide 11 : max1 < min2 or max2 < min1 is fulfilled, which formally proves that this axis is indeed a separating axis.

Note: For this constructed task, we can take a few shortcuts by reasoning based on the geometry of the situation. In a program intended to cope with any configuration of shapes, we would use the SAT test in the way it is implemented in the practical exercise.